in terms of the pressure and known thermodynamic quantities. If entropy and energy are considered functions of T and V, then

$$dS = \left(\frac{\partial S}{\partial V}\right)_T dV + \left(\frac{\partial S}{\partial T}\right)_V dT \tag{47}$$

and

$$dE = \left(\frac{\partial E}{\partial V}\right)_T dV + \left(\frac{\partial E}{\partial T}\right)_V dT \cdot \tag{48}$$

The combined first and second laws of thermodynamics when PdV work is considered is given by

$$TdS = dE + PdV \ .$$

On an isotherm, Eqs. (47) and (48) reduce to the following:

$$dS = \left(\frac{\partial S}{\partial V}\right)_T dV \text{ and } dE = \left(\frac{\partial E}{\partial V}\right)_T dV \ .$$

When these terms are substituted, the TdS equation becomes

$$T\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial E}{\partial V}\right)_T + P \ . \tag{49}$$

The term $(\partial S/\partial V)_T$ can be written with the aid of Maxwell's equations and the definitions for isothermal bulk modulus B_T and volume expansion coefficient β in the form

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial S}{\partial P}\right)_T\left(\frac{\partial P}{\partial V}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P\left(\frac{\partial P}{\partial V}\right)_T = \beta B_T \ .$$

Using this result, Eq. (49) can be expressed in terms of the dependant variable P and the material properties as

$$\left(\frac{\partial E}{\partial V}\right)_T = \beta B_T T - P \ . \tag{50}$$

The pressure and energy in Eq. (46) are regarded as functions of volume on a given isotherm and on the Hugoniot allowing the following to be written,

$$\left(\frac{\partial P}{\partial V}\right)_T = \frac{dP_T}{dV} , \left(\frac{\partial P}{\partial V}\right)_H = \frac{dP_H}{dV}$$